MAGNETO-OPTIC ROTATION AND ITS EXPLANATION BY A GYROSTATIC SYSTEM.[1]

II.

I MUST now endeavour to give some slight account of the theories that have been put forward in explanation of magneto-optic rotation. There is an essential distinction between it and what is sometimes called the natural rotation, the plane of polarised light produced by substances, such as solutions of sugar, tartaric acid, quartz, &c., some of which rotate the plane to the right, some to the left. When light is sent once along a column of any of those substances without any magnetic field, its plane of rotation is rotated just as it is in heavy glass or bisulphide of carbon in a magnetic field. But if the ray, after passing through the column of sugar or quartz, is received on a silvered reflector and sent back again through the column to the starting point, its plane of polarisation is found to be in the same direction as at first. Quite the contrary happens when the rotation is due to the action of a magnetic field. Then the rotation is found to be doubled by the forward and backward passage, and it can be increased to any required degree by sending the ray backward and forward through the substance, as shown in this other diagram (Fig. 8).

Thus the rotations in the two cases are essentially different, and must be brought about by different causes. In fact, as was first, I believe, shown by Lord Kelvin, the annulment of the turning in quartz, and the reinforcement of the turning in a magnetic field, produced by sending the ray back again after reflection at the surface of an optically denser medium, points to a peculiarity of structure of the medium as the cause of the turning of the plane of polarisation in sugar solutions and quartz, and to the existence of rotation in the medium as the cause of the turning in a magnetic field. Think of an elastic solid, highly incompressible and endowed with great elasticity of shape and of the same quality in different directions—a stiff jelly may be taken as an example to fix the ideas. Now let one portion of the jelly have bored into it a very large number of extremely small corkscrew-shaped cavities, having their axes all turned in the same direction. Let another portion have imbedded in it a very large number of extremely small rotating bodies, spinning-tops or gyrostats in fact, and let these be uniformly distributed through the substance, and have their axes all turned in the same direction.

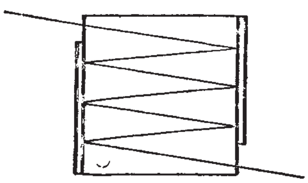

FIG. 8.

Both portions would transmit a plane-polarised wave of transverse vibration travelling in the direction of the axes of the cavities or of the tops with rotation of the plane of polarisation; but in the former case the wave, if reflected and made to travel back, would have the original plane of polarisation restored; in the latter the turning would be doubled by the backward passage.

To understand this it is necessary to enter a little in detail into the analysis of the nature of plane-polarised light. As I have already said, the elastic solid theory may not express the facts of light propagation, but only a certain correspondence with the facts. But its use puts this matter in a very clear way. In a ray of plane polarised light each portion of the ether has a motion of vibration in a line at right angles to the ray, and the direction of this line is the same for each moving particle. The lines of motion and the relative positions of the particles in a wave are shown in the first diagram (Fig. 1 p. 379). As the motion is kept up at the place of excitation, it is propagated out by the elastic resistance of the medium to displacement, and the configuration of particles travels outwards with the speed of light, traversing a wave-length (represented in the diagram by the distance between two particles of the row in the same phase of motion) in the period of complete to-and-fro motion of a particle in its rectilineal path.

Now, a to-and-fro motion such as this can be conceived as made up of two opposite uniform and equal circular motions. Think of two distinct particles moving in the two equal circles A B in this diagram (Fig. 9), with equal uniform speeds in opposite directions. Let each particle be at the top of its circle at the same instant; then at any other instant they will be in similar positions, but one on the right, the other on the left of the vertical diameter of the circle. Thus at that instant each particle is moving downward or upward at the same speed, while with whatever speed one is moving to the left, the other is moving with precisely that speed towards the right. Imagine now these two motions to be united in a single particle. The vertical motions will be added together, the right and left motions will cancel one another, and the particle will have a motion of vibration in the vertical direction of range equal to twice the diameter of the circles, and in the period of the circular motions.

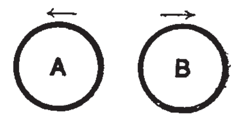

FIG. 9.

The rate of increase of velocity of the particle at each instant is the resultant obtained by properly adding together the accelerations of the particles in the circular motions, and therefore the force which must act on the particle to cause it to describe the vibratory motion just described is the resultant of the forces required to give to the two particles the circular motions which have just been considered.

Now, what we have done for any one particle may be conceived of as done for all the particles in a wave. To understand the nature of a wave in this scheme, we must think of a series of particles originally in a straight line in the direction of propagation of the ray, as displaced to positions on a helix surrounding that direction. Fig. A of this diagram (Fig. 10), regarded from the lower end, and the black spots on the model before you, show a left-handed helical arrangement. Let these particles be projected with equal speeds in the circular paths represented by the circle at the bottom of Fig. A. On this circle are seen the apparent positions of different particles in the helical arrangement when it is viewed by an eye looking upwards along its axis. This motion is shown by that of the black spots on the surface of the model (Fig. 11), when I set it into rotation about its axis. Let the particles be constrained to continue in motion exactly in this manner. As the model shows, the helical arrangement of the particles is displaced along the cylinder. This is the mode of propagation of a *circularly* polarised wave, which is made up of helical arrangements of particles which were formerly in straight lines parallel to the axis.

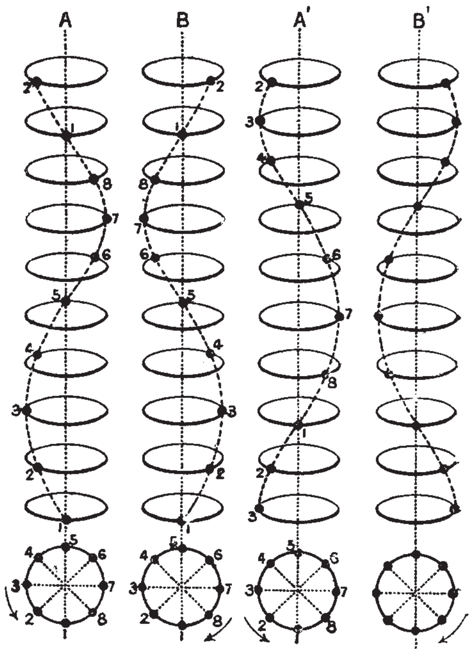

FIG. 10

The direction of propagation of the wave is clearly from the

[1] A discourse delivered at the Royal Institution by Prof. Andrew Gray, F.R.S. (Continued from p. 381.)

bottom of the diagram to the top, and from the end of the model towards your left to the other, when the particles have a right-handed motion, and is in the contrary direction when the direction of rotation is reversed. For a right-handed helical arrangement the direction of propagation for the same direction of motion of the particles is the opposite of that just specified. The direction of propagation remains, therefore, the same when the direction of motion and the helical arrangement of the particles are both reversed. All this can be made out from the diagram. Fig. B shows part of a right-handed arrangement of

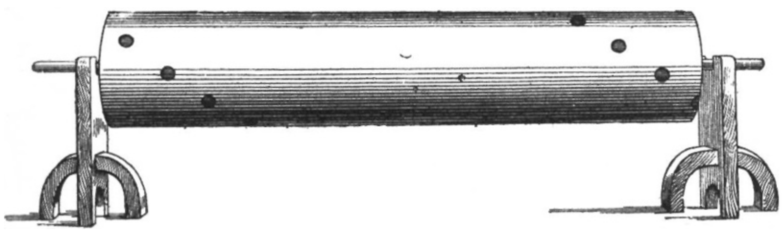

FIG. 11.

particles corresponding to the opposite arrangement of Fig. A; and if the particles have the motions shown at the bottom of the diagram the propagation will be for both in the same direction, from the bottom to the top.

In Fig. 10 we suppose the periods equal and also the wave-lengths, the distance along the axis from particle 1 to particle 9. The combination of the circular motions A and B gives rectilinear motion; the combination of the wave motions of Figs. A and B gives a plane polarised wave the plane of polarisation of which does not change in position. If, however, while the periods were equal, the wave-lengths were unequal as shown in this other diagram (Fig. 12), the plane of polarisation would rotate, as shown by the lines drawn across the paths in the figure on the right, for the circular motions of particles in the longer wave would gain on those in the shorter.

A little consideration will show that the direction of the resultant rectilinear motion will, in consequence of the unequal speeds of propagation, turn round as the wave advances, and will do so in the direction of motion of the particles in the more quickly travelling wave, generating the screw surface shown in the model I have already exhibited.

We must now consider the forces. The particles moving in the circular paths have accelerations towards the centres of these paths, and forces must be applied to them to produce these accelerations. These forces are applied in the present theory by the action of the medium, and it is the reactions of the particles on the medium that are properly called the centrifugal forces of the particles. The requisite centreward forces then are supplied by the state of strain into which the medium is thrown by the displacement of parts of it, which form in the undisturbed position a series of straight arrays in the direction of propagation, into these helical arrangements round that direction. The greater these elastic forces the greater the velocity of propagation of the wave.

In an elastic medium these forces depend on the *amount* of the relative displacements of the particles, and will be greater for displacements in the right-hand helical arrangement than for displacements in the opposite direction if the medium has a greater rigidity for right-handed distortion than for left, and the right-handed wave of distortion will be transmitted with greater speed, and *vice versâ*. This is the case of solutions of sugar and tartaric acid, quartz, &c., for which a helical structure has been supposed to exist in the medium.

Taking this case refer to Figs. A and B of our large diagram (Fig. 10), and let the right-handed wave travel the faster. Let the waves travel up, be reflected at the upper ends, as at the surface of a denser medium, and then travel down again. The reflected waves are those shown in Figs. A′, B′ of the diagram. By the reflection, the helical arrangement will be unaltered. But the plane of polarisation, as we have seen, turns round in space in the direction of the motion of the particles in the more quickly moving wave; it therefore turns round in the direction of the hands of a watch as the wave moves in the upward direction in the diagram, and in the opposite direction when the wave is travelling back. Thus the rotation of the plane of polarisation produced in the forward passage is undone in the backward.

It is easy to see that the same thing will take place if the reflection is at the surface of an optically rarer medium, so that the direction of motion of the particles is the same in the reflected as in the direct wave. The helical arrangements, however, are reversed by the reflection, and hence the wave which travelled the more quickly forward travels the more slowly back, and again the turning of the plane of polarisation is annulled by the backward passage. Thus Lord Kelvin's hypothesis of difference of structure completely explains the phenomena.

We pass now to the other case, that of magneto-optic rotation. Let us suppose, to fix the ideas, that the right-handed circular ray travels faster than the other, and that whether direct or reversed. Here, as in the other case, the elastic reaction of the medium on the displaced particles depends only on the distortion, and if there be no structural peculiarity in the medium there must be the same reaction in the particles in both the circular waves which combine to make up the plane-polarised one.

Thus the actions on the particles being the same for both waves, and the velocities of propagation being different, the motions concerned in the light propagation cannot be the same. There must in fact be a motion already existing in the medium which, compounded with the motions concerned in light propagation, give two motions which give equal reactions in the medium against the equal elastic forces, applied to the particles in the case of equal helical displacements.

Thus Lord Kelvin supposes that in the medium in the magnetic field there exists a motion capable of being compounded

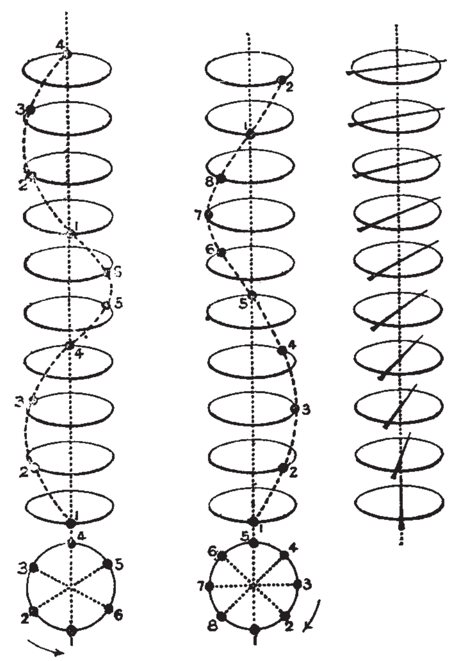

FIG. 12.

with the luminiferous motion of either circularly polarised beam. The latter is thus only a component of the whole motion.

In the very important paper in which he has set forth his theory Lord Kelvin expresses his strong conviction that his dynamical explanation is the only possible one. If this view be correct, Faraday's magneto-optic discovery affords a demonstration of the reality of Ampère's theory of the ultimate nature of magnetism. For we have only to consider the particles of a magnetised body as electrons or groups of charges of electricity, ultimate as to smallness, rotating about axes on the whole in

alignment along the direction of the magnetic force, and with a preponderance of one of the two directions of rotation over the other. Each rotating molecule is an infinitesimal electromagnet, of which the current distribution is furnished by the system of convection currents constituted by the moving charges.

The subject of magneto-optic rotation has also been considered by Larmor, and two types of theory of these effects have been indicated by him in his report on the "Action of Magnetism on Light." One is represented by Lord Kelvin's theory, which is illustrated by Maxwell's chapter on molecular vortices in his "Electricity and Magnetism." FitzGerald's paper "On the Electromagnetic Theory of the Reflection and Refraction of Light," in the *Philosophical Transactions* for 1880, is related to Maxwell's theory, and explains the rotation produced by reflection from the pole of a magnet by means of the addition of a term to the energy of the system. The other theory is also a purely electromagnetic one, and supposes that the effects are due to a kind of æolotropy of the medium set up by the magnetisation, and so attributes them to a change of structure which introduces rotational terms into the equations connecting *electric* displacements and electric forces. This latter theory therefore regards the magneto-optic rotation as only a secondary effect of the magnetisation, which is not supposed to exert any direct dynamical influence on the transmission of the light-waves.

It is not possible here to enter into the subject of these theories, but I should like to direct attention to a paper by Mr. J. G. Leathem, published in the *Philosophical Transactions*, in which the type of theory just referred to has been worked out and compared in its results with the experiments of Sissingh and Zeeman in reflection. These investigators made measurements of the phase and amplitude of the magneto-optic component of the reflected light for various angles of incidence. For both these quantities the theoretical results of Leathem agree very well with the observed values.

Returning now to the gyrostatic medium, between which and the electromagnetic theory, it is to be remembered, there is a correspondence, we may inquire in what way the gyrostats, when moved by the vibrations of the medium, react upon it, and so affect the velocity of propagation. The motion of a gyrostat is often regarded as mysterious, and it can hardly be fully explained except by mathematical investigation. But the general nature of its action may be made out without much difficulty. First of all, a gyrostat consists of a massive fly-wheel running on bearings attached to a case which more or less completely encloses the wheel. The mass of the wheel consists in the main of a massive rim, which renders as great as possible what is called the moment of momentum of the wheel when rotating about its axis. The diagram (Fig. 13) represents a partial section of the case and fly-wheel of a gyrostat, showing the arrangement of fly-wheels and bearings.

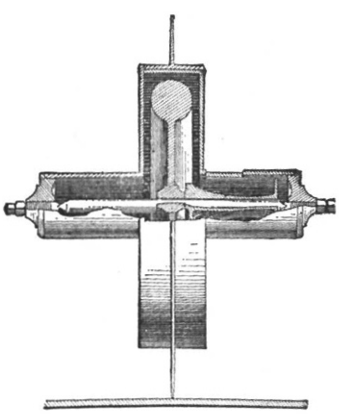

FIG. 13.

Now let the fly-wheel of such a gyrostat be rapidly rotated, and the gyrostat be hung up as shown in this other diagram (Fig. 14), with the plane of the fly-wheel vertical, and a weight attached to one extremity of the axis. The gyrostat is not tilted over, but begins to turn round the cord by which it is suspended with a slow angular motion which is in the direction of the horizontal arrow, if the direction of rotation is that of the circular arrow shown on the case. The same thing is shown by the experiment I now make. I spin this gyrostat and hang it with the axis of rotation horizontal by passing a loop of cord round one end of the axis so that the weight of the gyrostat itself forms the weight tending to tilt it over about the point of suspension. The axis of rotation here again remains nearly horizontal, but turns slowly round in a horizontal plane as before.

The explanation in general terms is this. The weight gives a couple tending to turn the gyrostat about a horizontal axis at right angles to that of rotation. This couple in any short interval of time produces moment of momentum about the axis specified, the amount of which is the moment of the couple multiplied by the time, and may be represented in direction and magnitude by the line OB. This must be compounded with the moment of momentum OA already existing about the axis of rotation, and gives for the resultant moment of momentum the line OC, which is the direction of the axis of rotation after the lapse of the short interval of time. The axis of rotation thus turns slowly round in the horizontal plane, and the more slowly the more rapidly the fly-wheel rotates.

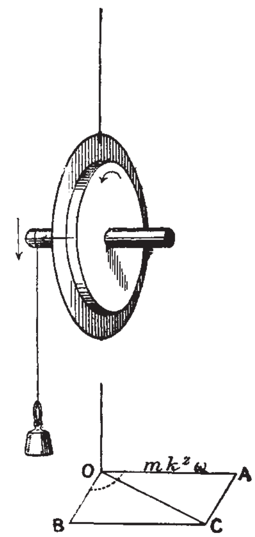

FIG. 14

The gyrostat in fact must have this precessional motion, as it is sometimes called, in order that the moment of momentum of the gyrostat about a vertical axis may remain zero. That it must remain zero follows from the fact that there is no couple in a horizontal plane acting on the gyrostat.

Thus any couple tending to change the direction of the axis in any plane produces a turning in a perpendicular plane. For example, if a horizontal couple, that is about a vertical axis, were applied to the axis of the gyrostat in the last figure it would turn about a horizontal axis, that is, would tilt over.

Again, consider a massive fly-wheel mounted on board ship on a horizontal axis in the direction across the ship. The rolling of the ship changes the direction of the axis, and produces a couple applied by the fly-wheel to the bearings and an equal and opposite couple applied by the bearings to the fly-wheel. This couple is in the plane of the deck, and is reversed with the direction of rolling, and has its greatest value when the rate of turning of the ship is greatest. Thus the force on one bearing is towards the bow of the ship, the force on the other towards the stern, during a roll from one side to the other; and these forces are reversed during the roll back again. This is the gyrostatic couple exerted on its bearings by the armature of a dynamo on shipboard.

In the same way, when a gyrostat is embedded in a medium and the medium is moving so as to change the direction of the axis of rotation, a couple acting on the medium in a plane at right angles to the plane of the direction of motion is brought into play. To fix the ideas, think of a row of small embedded gyrostats along this table with their axes in the direction of the row, and their fly-wheels all rotating in the same direction. Now let a wave of transverse displacement of the medium in the vertical direction pass along the medium in the direction of the chain. The vibratory motion of each part of the medium will turn the gyrostatic axis from the horizontal, and thereby introduce horizontal reactions on the medium. Again, a wave of horizontal vibratory motion will introduce vertical reactions in the medium from the gyrostats.

Now, a wave of circular vibrations, like those we have already considered, passing through the medium in the direction of the chain, could be resolved into two waves of rectilinear vibration, one in which the vibration is horizontal, and another in which the vibration is vertical, giving respectively vertical and horizontal reactions in the medium. The magnetisation of the medium is regarded as due to the distribution throughout it of a multitude of rotating molecules, so small that the medium, notwithstanding their presence, seems of uniform quality. The molecules have, on the whole, an alignment of their axes in the direction of magnetisation. These reactions on the medium when worked out give terms in the equations of wave propagation of the proper kind to represent magneto-optic rotation.

It is worthy of mention that the addition of such terms to the equation was made by McCullagh, the well-known Irish mathematician, who, however, was unable to account for them by any

physical theory. The necessary physical theory may be regarded as afforded by the mechanism which thus forms an essential part of Lord Kelvin's mode of accounting for magneto-optic effects.

Lord Kelvin, in his Baltimore Lectures, has suggested for magneto-optic rotation a form of gyrostatic molecule consisting, as shown in the figure, of a spherical sheath enclosing two equal gyrostats. These are connected with each other and with the case by ball-and-socket joints at the extremities of their axes, as shown in Fig. 15. If the spherical case were turned round any axis through the centre no disalignment of the gyrostats contained in it would take place, and it would act just like a simple gyrostat. If, however, the case were to undergo translation in any direction except along the axis, the gyrostats would lag behind, and the two-link chain which they form would bend at the centre. This bending would be resisted by the quasi-rigidity of the chain produced by the rotation, and the gyrostats would react on the sheath at the joints with forces as before at right angles to the plane in which the change of direction of the axis takes place.

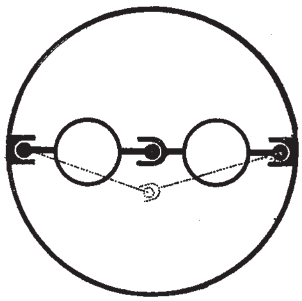

FIG. 15.

The general result is, that if the centre of this molecule be carried with uniform velocity in a circle in a plane at right angles to the line of axes, the force required for the acceleration towards the centre, and which is applied to it by the medium, is greater or less according as the direction in which the molecule is carried round is with or against the direction of rotation of the gyrostats. That is, the effect of the rotation is to virtually increase the inertia of the molecule in the one case and diminish it in the other.

These molecules embedded in the medium are supposed to be exceedingly small, and to be so distributed that the medium may, in the consideration of light propagation, be regarded as of uniform quality.

Lord Kelvin's last form of molecule, it may be pointed out, if the surface of its sheath adheres to the medium, will have efficiency as an ordinary single gyrostat as regards rotations of the molecule, and efficiency likewise as regards translational motion of the centre of the molecule. The former efficiency can be made as small as may be desired by making the molecule sufficiently small; the latter may be maintained at the same value under certain conditions, however small the molecule be made.

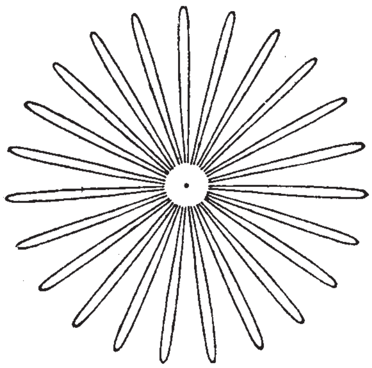

FIG. 16.—Path of the Bob of a Gyrostatic Pendulum. As the pendulum moves, it passes from one ray to another on the opposite side, and the direction of motion at each swing alters through the angle between two rays. The central parts of the rays are left out. The marking point does not pass exactly through the centre.

The lately discovered effect of a magnetic field in giving one period of circular oscillation of a particle or another according as the particle is revolving in one direction or the other about the direction of the magnetic force, is connected with magneto-optic rotation. There is a connection between velocity of propagation and frequency of vibration, which is exemplified by the phenomena of dispersion. In the Faraday effect, the two modes of vibration, if of the same period, have different velocities of vibration, consequently these two modes of vibration must have different frequencies for the same velocity of propagation.

The vibrations of the molecules of a gas in which the Zeeman effect is produced by a magnetic field may be represented by the motion of a pendulum the bob of which contains a rapidly rotating gyrostat with its axis in the direction of the supporting wire of the pendulum. The period of revolution of the bob when moving as a conical pendulum is greater or less than the period when the gyrostat is not spinning according as the direction of revolution is against or with the direction of rotation.

The bob when deflected and let go moves in a path which constantly changes its direction, so that if a point attached to the bob writes the path on a piece of paper, a star-shaped figure is obtained. I cause the gyrostatic pendulum here suspended to draw its path by a stream of white sand on the black board placed below it, and you see the result.

I must here leave the subject, and may venture to express the hope that on some other occasion some one more specially acquainted with the electromagnetic aspects of the phenomenon may be induced to place the latest results of that theory before you.

UNIVERSITY AND EDUCATIONAL INTELLIGENCE.

MR. JAMES BROWN THOMSON, of Kinning Park, Glasgow, who died ten months ago, left 80,000*l.* to Glasgow institutions —mostly educational and benevolent. The Glasgow University will receive 10,000*l.*

THE recent discussion in NATURE on "The Duties of Provincial Professors" forms the subject of a short critique in the August number of the *Educational Review.* While fully endorsing the general views expressed in our columns, the *Review* remarks: "There is only one flaw in the indictment —the insinuation, namely, that university professors should take no part in the social life and physical activities, the general discipline, the corporate existence of the university or university college." But where does this flaw exist? No such insinuation is made in the article in NATURE.

THE Department of Science and Art has issued the following list of successful candidates for Royal Exhibitions, National Scholarships, and Free Studentships (Science) awarded this year:—Royal Exhibitions: William M. Selvey, Edward C. Moyle, Archibald D. Alexander, Charles W. Price, George F. A. Cowley, Edgar Sutcliffe, Sydney A. Edmonds. National Scholarships for Mechanics: Francis P. Johns, George F. Turner, Walter A. Scoble, Arthur J. Spencer, William H. Adams. Free Studentships for Mechanics: R. Borlase Matthews, William H. Outfin. National Scholarships for Physics: William R. Daniel, William J. Lyons, James Lord, William M. Varley, Wilfred H. Clarke. Free Studentships for Physics: John H. Shaxby, Gerald Henniker. National Scholarships for Chemistry: William D. Rogers, John H. Crabtree, Howard E. Goodson, Arthur H. Higgins, Montague W. Stevens. Free Studentships for Chemistry: John R. Horsley, Arthur C. Nicholson. National Scholarships for Biology: Eric Drabble, Louis E. Robinson, Ernest A. Wraight, Reginald F. G. Bayley, Harold B. Fantham. National Scholarships for Geology: William H. Goodchild, Thomas Thornton.

THE following list of candidates, successful in this year's competition for the Whitworth Scholarships and Exhibitions, has been received from the Department of Science and Art:—Scholarships, tenable for three years, 125*l.* a year each: Alec W. Quennell, London; Hanson Topham, Great Horton, Bradford; William V. Shearer, Langside, Glasgow; George Wall, Oldham. Exhibitions, tenable for one year, value 50*l.* each: Arthur J. Spencer, Portsmouth; George F. Turner, Sheffield; Harold P. Philpot, London; William H. Adams, Devonport; Edward C. Moyle, Devonport; Walter A. Scoble, E. Stonehouse, Devon; Archibald D. Alexander, Portsmouth; Sydney A. Edmonds, Devonport; George F. A. Cowley, Portsmouth; Albert Wilson, Leeds; Edwin J. Britton, Portsmouth; Harry Duncan, Plumstead; Samuel C. Rhodes, Morley, Leeds; Harry M. Andrew, Manchester; Alexander P. Traill, North Shields; Leonard Bairstow, Halifax; William T. S. Butlin, Bristol; Albert E. Dodridge, Devonport; James Lowe, Alloa; William J. Rodd, Plumstead; Francis C. Rendle, Plymouth; Thomas E. Heywood, Cardiff; James Paul, Woolwich; Charles P. Raitt, Portsmouth; Charles H. Booth, Bolton; Edward Howarth, Oldham; Percy Down, London; Marshall H. Straw, Sneinton, Nottingham; R. Borlase Matthews, Swansea; Samuel Crossley, Oldham.